

Examples

Ex(1) A card is drawn from a well shuffled pack of playing cards. Find the probability that it is either a diamond or a king.

Solution: Let A denote the event of drawing a diamond.
 B denote the event of drawing a king.

$$P(A) = \frac{13}{52} = \frac{1}{4} \quad P(B) = \frac{4}{52} = \frac{1}{13}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = \frac{1}{4} + \frac{1}{13} - P(A \cap B)$$

Probability of drawing either king or diamond

There is only one case favourable to the event $A \cap B$ (king of diamond). Hence, $P(A \cap B) = \frac{1}{52}$

$$\Rightarrow P(A \cup B) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} \\ = \frac{13+4-1}{52} \\ = \frac{16}{52} \\ = \frac{4}{13}$$

Ex(2) A pair of dice is rolled. If the sum on the two dice is 9. Find the probability that one of the dice is showed 3.

Solution:

Let A = The sum on two dice is 9
 B = One dice shows 3

In a random toss of two dice.

$$S = \{1, 2, \dots, 6\} \times \{1, 2, \dots, 6\} \\ n(S) = 6 \times 6 = 36.$$

Event	Favourable cases	Probability
A	(3,6) (6,3) (4,5) (5,4)	$P(A) = \frac{4}{36} = \frac{1}{9}$
$A \cap B$	(3,6) (6,3)	$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/18}{1/9} = \frac{1}{2} = 0.5$ (Ans)

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Ex(3) Choose the correct alternative For two equally likely, exhaustive and independent events A and B, $P(AB)$ is:

(i) 0, (ii) 0.25 (iii) 0.50 (iv) 1.

Solution

As A and B are equally likely

$$P(A) = P(B)$$

As A and B are exhaustive

$$P(A) + P(B) = 1$$

$$P(A) = P(B) = \frac{1}{2}$$

As A and B are independent also

$$P(A \cap B) = P(A)P(B)$$

$$= \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4} = 0.25$$
 (Ans)

Ex(4) A box contains 3 red and 7 white balls. One ball is drawn at random and its place a ball of the other colour is put in the box. Now one ball is drawn at random from the box. Find the probability that it is red.

Solution

Let

A = Drawing a white ball in the first draw.

B = Drawing a Red ball in the first draw.

C = Drawing a Red ball in the second draw.

$$P(A) = \frac{7}{10} \quad P(B) = \frac{3}{10}$$

The required event of getting a red ball in the 2nd draw can materialise in the following mutually exclusive ways:

- (1) First draw gives a white ball and then red ball is drawn in the 2nd draw ($A \cap C$)
- (2) First draw gives a red ball and then red ball is drawn in the 2nd draw ($B \cap C$)

According to Addition theorem of probability

$$\begin{aligned} P(1) + P(2) &= P(A \cap C) + P(B \cap C) \\ &= P(A) P(C|A) + P(B) \cdot P(C|B) \\ &= \frac{7}{10} P(C|A) + \frac{3}{10} P(C|B) \end{aligned}$$

$P(C|A) = P(\text{Drawing red ball in 2nd draw when the first ball drawn is white})$

\Rightarrow if first ball is drawn white, then it is replaced by the ~~red~~ red ball (the other colour ball), then the box for 2nd draw becomes 4 red and 6 white balls.

$$P(C|A) = \frac{4}{10}$$

Similarly $P(C|B) = \frac{2}{10}$

Required Probability,

$$= \frac{7}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{2}{10}$$

$$= \frac{28 + 6}{100}$$

$$= \frac{34}{100} = 0.34 \text{ (Ans)}$$

Reference: Fundamentals of Statistics by S. C. Gupta.